If these be transformed by S_2, the number, ρ, remaining invariant, satisfies the congruence $3-\rho\equiv 0 \pmod 3$, and since $S_2^{-1}\{S_3\}S_2=\{S_3\}$, we have $\rho=3$, and $S_2S_4=S_4S_2$. Suppose next that $S_4^{-1}S_1S_4=S_1^xS_2^y$.

$$(S_3S_4)^{-1}S_1(S_3S_4)=S_1^{-x}S_2^{-y}=(S_4S_3)^{-1}S_1(S_4S_3)=S_1^{-x}S_2$$
$$-2y\equiv 0 \pmod 3)\,;\ y\equiv 0 \pmod 3),$$

and as S_4 is not permutable with $S_1 x\equiv -1 \pmod 3$. So that $G=\{S_3, S_4, S_1\}\{S_2\}$ is defined by the relations

$$S_1^3=S_2^3=S_3^2=1,\quad S_1S_2=S_2S_1,\quad S_3S_4=S_4S_3,\quad S_3^{-1}S_1S_3=S_1^{-1},\quad S_2S_3=S_3S_2,$$
$$S_4^{-1}S_1S_4=S_1^{-1},\quad S_2S_4=S_4S_2.$$

The subgroups $\{S_3, S_4, S_1\}$ will be recognized as the abstract form of the tetrahedron rotation group.

TESTS OF DIVISIBILITY BY 7, 13, AND 17.

By MISS ALICE CHURCH, New York City.

1. Seven is an exact divisor of any number of which the units figure doubled differs frcm the number represented by the remaining digit or digits by zero or by a multiple of seven. Thus, in the number 14, twice 4 are 8; 1 from 8 leaves 7.

In 168, twice 8 are 16; 16 from 16 leaves 0.

In 532, twice 2 are 4; 4 from 53 leaves 49.

Corollary A. When the remainder after subtraction is 0, the original number is a multiple of 3 as well as of 7, therefore is a multiple of 21.

When the number to be treated is so large that the remainder found by subtracting the doubled unit from the rest of the number is too large to be judged by inspection, the same test may be applied to the remainder as to the original number. This process may be repeated until a remainder be found which is small enough to be factored by inspection.

For example, 22134; twice 4 are eight; 8 subtracted from 2213 leaves 2205; twice 5 are 10; 10 from 220 leaves 210; 210 at once appears as a multiple of 7.

Demonstration. Multiplying the units figure by 2 and placing the product in the tens column and ignoring the units figure in the subtraction is really multiplying the units figure by 21, which is a multiple of 7. The test, then, becomes merely the subtraction of a multiple of seven from a possible multiple of 7, or vice versa, and as the difference between multiples must be a multiple, the number tested is divisible by 7 if the difference found is a multiple of 7.

The value of the test rests upon the facility with which the multiple of 21 is created, inasmuch as to multiply by 2 is a mental process much more surely within the mind of a child than dividing by 7.

2. A test for the divisibility by 13 is to be found by a similar process and upon the same principles—being based on the fact that 91 is a multiple of 7. Multiply the unit figure by 9 and find the difference between the product and the number without its unit figure. Thus, in 1183, $9\times3=27$, $118-27=91$, and in 91, $9\times1=9$, $9-9=0$. For 325, $9\times5=45$, $45-32=13$.

3. Likewise for 17, multiply by 5, since 51 is a multiple of 17. So for 595, $5\times5=25$; $59-24=34$. For 2244, $5\times4=20$, $224-20=204$; $5\times4=20$, $20-20=0$.

NOTE ON THE EVOLUTE OF AN ALGEBRAIC CURVE.

By A. H. WILSON, Instructor of Mathematics, University of Illinois.

The following method of forming the evolute of an algebraic curve may be of interest.

Let $f(x, y)=\varphi$ represent the curve, and $y-y_1=l(x-x_1)$ its normal at the point (x_1y_1) on the curve, l being a function of x_1 and y_1. The elimination of x_1 (or y_1) between $f(x_1, y_1)=0$ and $\beta-y_1=l(\alpha-x_1)$, gives an equation

$$\varphi(y_1)=0 \text{ (or } \psi(x_1)=0),$$

whose roots are the ordinates (or the abscissas) of the points on the curve the normals at which pass through the point (α, β).

The evolute may be regarded as the locus of points from which two of the normals through (α, β) to the curve are coincident; and hence the equation of the evolute is the relation between α and β obtained by setting equal to zero the discriminant of $\varphi=0$ (or $\psi=0$).

The application of the method is obviously very limited.

DETERMINATION OF THE RADIUS OF CURVATURE OF THE CYCLOID WITHOUT THE AID OF THE CALCULUS.

By FREDERIC R. HONEY, Hartford, Conn.

Let A represent any regular polygon. If we roll it along the straight line BC into the positions A', A'', ... bringing each side in succession into coinci-